Applications of the Kelly Criterion in Finance and Sports

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We discuss uses of the Kelly Criterion for investing in stocks and other financial instruments and in betting on sports. Real examples of investors such as Warren Buffett and Ed Thorp (20% return over 25 years) are given. Ray Stefani explains the applications to sports betting. He has achieved a return of 15% over several years

The Kelly Criterion

- In probability theory The Kelly criterion is a formula for choosing the sizes of bets that leads almost surely to higher wealth than any other
  strategy in the long run (as the number of bets approaches infinity).

- This is done by maximizing the expected value of the natural logarithm of wealth, as opposed to maximizing the wealth.

- It started with work by J. K. Kelly, a researcher at Bell Labs, in 1956.

The Kelly Criterion

- It has been used successfully by investors such as Edward Thorp (author of Beat the Dealer and Beat the Market) , Warren Buffett, and Bill Gross.
- It is also used in sports such as soccer and rugby (by Ray Stefani), and horse racing.
- The formula is f=(p(b+1)-1)/b, where
  f=how much to bet, as a fraction of the bankroll
  b=the net fractional odds received on the bet, and
  p=the probability of a win (q=1-p is the probability of a loss).

The Kelly Criterion

- Example: Suppose you have a 60% chance of winning (p=0.60) and b=1.
- Then f=(0.60(2)-1)/1=0.20. Thus the gambler should bet 20% of the bankroll every time to maximize the long-run growth of the bankroll.
- If p=q=0.50, then f=0, so the gambler should bet nothing.
- If p<q, then f is negative, so the gambler should not play unless he or she can place a bet “against the house”.

The Kelly Criterion

- Consider the game of American roulette. The gambler can place a bet with b=1 on red.
- There are a total of 18 red numbers and 20 non-red numbers. Then f=[(18/38)(2)-1]/1=-1/19.
- The gambler would then bet one-nineteenth of their bankroll that red will not come up. However, this is not allowed in the casinos, so, in general, the gambler should not play.
- Ed Thorp has devised a system, using probability and physics, in which the gambler in theory can win if the roulette wheel is balanced (p=-1/19) or unbalanced).

The Kelly Criterion Properties

- For even money bets, with b=1, f =2p-q.
- Using Kelly the gambler will never go broke.
- The absolute amount bet is absolutely increasing in wealth.
- The bets may be a large fraction of the gambler’s current wealth, when the wager is favorable and the risk of loss is small.

Size Matters

The Journal of Portfolio Management

INVITED EDITORIAL COMMENT

Rational Decision Making under Uncertainty: *Observed Betting Patterns on a Biased Coin*

VICTOR HAGHANI AND RICHARD DEWEY

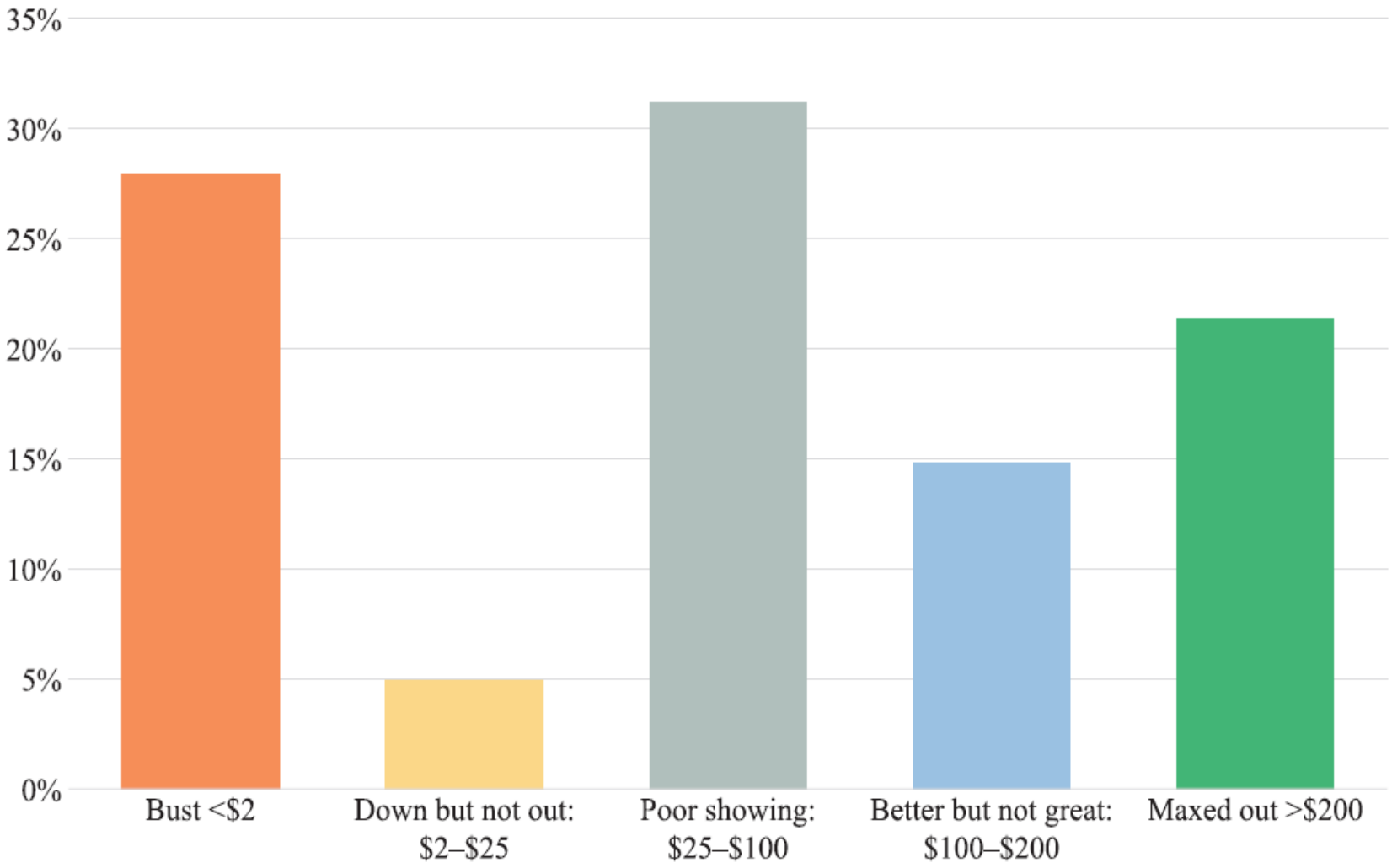
You're invited to a talk by a former hedge fund manager who was a partner at a fund that famously flopped about 20 years ago. You turn up, hoping to hear some valuable insights or at least some entertaining tales, but instead you are offered a stake of $25 to take out your laptop and bet on the flip of a coin for 30 minutes. You're told that the coin is biased with a 60% probability of coming up heads, and you can bet as much as you like on heads or tails on each flip. You will be given a check for whatever amount is in your account at the end of the half hour.[1]

That's it. Would you feel it was worth your time to play—or would you walk out? How would you play the game? What heuristic or mental tool kit would you employ? These questions led us to conduct this exact experiment. By having participants engage in an activity as simple as flipping a coin, we can easily isolate and observe the betting strategy and its evolution. This simple game also turns out to have similar properties to

THE EXPERIMENT

Our coin-flipping experiment was played by 61 subjects, in groups of 2–15, in the quiet setting of office conference rooms or university classrooms. The experiment began when subjects were directed to a URL that contained a purpose-built application for placing bets on the flip of a simulated coin. Prior to beginning, participants read a detailed description of the game, which included a clear statement, in bold, indicating that the simulated coin had a 60% chance of coming up heads and a 40% chance of coming up tails. Participants were given $25 of starting capital, and it was explained both in text and verbally that they would be paid, by check, the amount of their ending balance subject to a maximum payout. The maximum payout would be revealed if and when subjects placed a bet that if successful would make their balance greater than or equal to the cap. We set the cap at $250, ten times the initial stake. Participants were told that they could play the game for 30 minutes; and

Summary of Coin Flipper Performance
Betting on a Coin with Disclosed Bias Towards Heads of 60%
$25 Starting Stake, $250 Maximum Payout



Mistakes Made

- Betting on Tails
- Under-betting
- Erratic Betting
- Over-betting

General Form of Optimal Betting / Investing Strategy

Formulated in 1956 by John Kelly

Information arrives over a noisy channel (but the horse gambler has some edge)

Always bets on a positive edge (never bet tails)

Bet consistent fraction of wealth

Vary that fraction with gains and losses

Key idea: link the edge to the bet amount

Properties of Kelly

- Optimization Target = Constraint
  - Most often this is money. It means you can’t bet more than you have.
- All bets are independent
- Only works in the long run
  - Aaron Brown paper
- Finite variance (not fat tails)
- Assumptions on risk aversion levels
  - Assumes log utility
  - These can be modified

Kelly Summary

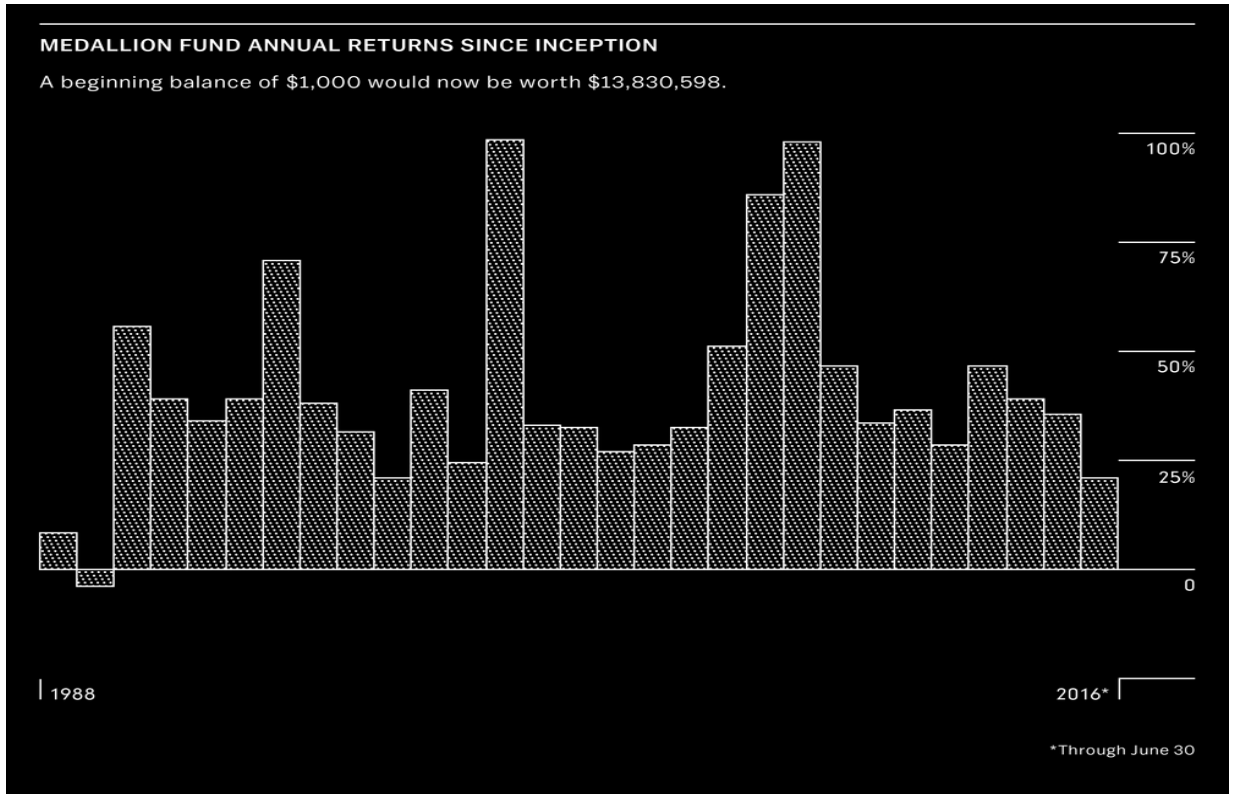
- Avoid Gamblers Ruin
  - Playing a positive expected games
  - Finite vs infinite wealth
- Don’t need utility functions
- Maximize wealth, subject to risk levels
  - Helps balance risk and reward

Renaissance Technologies

Markets Magazine

Inside a Moneymaking Machine Like No Other

The Medallion Fund, an employees-only offering for the quants at Renaissance Technologies, is the blackest box in all of finance.



"If you gave an investor the next day's news 24 hours in advance, he would go bust in less than a year."

- Nassim Taleb

Using the Kelly Criterion for Sports Betting

- Two important parameters are essential if we want to use the Kelly Criterion to bet on sports.

- First and foremost, the bettor needs an accurate and independent value of the probability (P) of a bet being successful. Accurately modeling and predicting a team or individual's win probabilities is a hugely important capability.

- Second, the bettor needs a simple measure of the betting odds.

Betting Odds

- **Fractional odds (F)** and **Decimals odds (D)** are the two most commonly used forms for bookmaker odds. **F** must be either be converted to **D** or, better yet, get **D** from the bookmaker in the first place.

- **Fractional odds** are written in the format profit:bet or profit/bet.
  3:2 means a profit more than the bet (bet on the underdog).
  2:5 means a profit less than the bet (bet on the favourite).
  This format is too complicated to insert into the Kelly Criterion.

- **Decimal odds** are written as one decimal number = (profit + bet)/bet.
  3:2 becomes **D** = (3+2)/2 = 2.5. (>2 is an underdog bet).
  2:5 becomes **D** = (2+5)/5 = 1.4. (<2 is a favourite bet).

The Kelly Criterion

- In 1956, Kelly derived the fraction (**F**) of a bankroll to wager on a bet for maximum exponential return on capitol, depending on **P** and **D**:
  F = (PD – 1) / (D-1) = (Expected Profit) / (Possible Profit)
- To make a profit, PD must be >1, therefore P>1/D. For the examples:
  for D = 2.5 (for an underdog) P must be more than 0.4,
  and for D = 1.4 (for a favourite) P must be more than 0.71.

How the Kelly Criterion Makes Good Sense

- As Kelly derived it for exponential growth of capital, the fraction of a bankroll to bet is given **F = (PD – 1) / (D-1).**
- In general, an increase in the probability P of winning causes the bet to increase as a fraction of the bankroll. That makes intuitive sense because the more likely the bet is to be won, the more should be the bet.
- In general, an increase in the decimal odds D causes a reduction in the fraction of the bankroll to be bet. That also makes intuitive sense because when D increases, the probability of picking that team to win decreases.

What I Predict

Home Team	Away Team	Probabilities/ Fair Odds			Goal Difference/ Total	Home	Away
		HW	D	AW			
Aston Villa	**Newcastle**	**0.47**	**0.18**	**0.36**	**0.3**		
		2.15	**5.64**	**2.80**	**2.5**	**1**	**1**
Brighton	Southampton	0.28	0.24	0.48	-0.3		
		3.58	4.20	2.07	2.6	1	1
Burnley	Everton	0.21	0.21	0.58	-0.7		
		4.68	4.82	1.73	3.1	1	2
Chelsea	Leeds	0.56	0.16	0.29	0.8		
		1.80	6.30	3.50	3.0	2	1

My Adjustments to Kelly for Sports Betting

- I found out in early work, that F was a bit too high in many cases. I use ½ Kelly so **F = 0.5 (PD – 1) / (D-1).**
- Do not bet when D< 1.5 (not enough profit).
- Do not bet when D>4.3 (winning bets less than 23% of the time).
- Do not bet more than 10% of the bankroll (too much risk on one bet).
- Do not bet less than 1.5% of the bankroll (too small to be worthwhile).
- To be risk averse for a week's bets, do not exceed 25% of the bankroll.
- If there is only one Kelly bet in a set of games, do not bet because good and bad results cannot offset.

Average Profit on Turnover Was 15% for 28 Seasons in 3 Sports

- As the following table shows, by combining seasonal averages of the ratio of money won divided by total betting turnover, an average profit on turnover of 15% resulted. Given that the weekly bet was limited to about 20-25% of the bankroll, there was about a 3% growth of that bankroll per week which resulted in significant return on the investment of that bankroll, especially for soccer with the longest season.

Average Profit on Turnover Was 15% for 28 Seasons in 3 Sports

Sport	NFL (American Professional Football)	Soccer (English Premier League)	Rugby Union: Zurich and Guinness (England); Super Rugby (Australia, New Zealand and South Africa)	Total
Seasons	2	6	20	**28**
Games Evaluated	318	1170	1603	**3091**
% Games Bet	49.7%	**31.2%**	39.9%	**37.6%**
% Bets Won	39.2%	**44.7%**	**47.7%**	**45.6%**
Profit on Turnover	12.5%	**19.1%**	14.2%	**15.4%**
Return on Investment	43.3%	**72.5%**	39.3%	**46.7%**